

MATH 2050A Tutorial 3

1. Evaluate the limit

$$\lim_n \frac{5n^2 + 2n + 1}{3n^2 + n + 2}$$

by definition.

2. Let $A \subseteq \mathbb{R}$. Suppose $\sup A = \alpha \in \mathbb{R}$.

(a) Show that there exists a sequence (a_n) in A converging to α .

(b) Show that there exists a monotone increasing sequence (b_n) in A converging to α .

3. Let $a > 0$. Show that

$$\lim_n \frac{a^n}{n!} = 0$$

4. Let $p \in \mathbb{N}$ and $b \in \mathbb{R}$ satisfy $0 < b < 1$. Show that $\lim(n^p b^n) = 0$.

5. Let (x_n) be a sequence of positive real numbers. Suppose $\lim \sqrt[n]{x_n} = L$, where L is a non-negative real number.

(a) If $0 \leq L < 1$, show that $\lim x_n = 0$.

(b) If $L > 1$, show that (x_n) is divergent.

(c) What happens if $L = 1$?

6. Let (x_n) be a sequence of positive real numbers. Suppose $\lim x_{n+1}/x_n = L$, where L is a non-negative real number. Show that $\lim \sqrt[n]{x_n} = L$.